BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B71 02032

SUBJECT: The Dynamics and Performance of a Four-Wheel Vehicle - Case 320

DATE: February 16, 1971

FROM: S. Kaufman

ABSTRACT

Equations of motion have been formulated for a four-wheel vehicle as it traverses a surface characterized by slopes, bumps, craters, or a power spectrum. A primitive control law to guide the vehicle to a predetermined destination is included. Also included is a propulsion algorithm for independent series motor drives.

These equations and algorithms have been incorporated into the digital program ROVER, which monitors among other things power consumption, distance traveled, velocities, accelerations, and wheel forces. A sample problem is included of the Lunar Roving Vehicle negotiating short traverses over a random topography propelled by series motors. Computer runs were made on three terrains: a rough mare, a smooth mare, a perfectly smooth terrain. Preliminary results indicate a small increase (∿3%) in energy consumption when going from a perfectly smooth terrain to the smooth mare but a significant increase in energy consumption (∿30%) when going from the smooth mare to the rough mare.

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MEMORANDUM FOR FILE

INTRODUCTION

This report is an extension to reference 1, which presented the equations of motion of a four-wheel vehicle with independent suspension, as it traverses a defined surface characterized by slopes, bumps, and craters. The impetus for that study was to ascertain the dynamic stability of the Lunar Roving Vehicle (LRV) on the moon's surface. The governing equations of motion have been modified slightly from that of reference 1 and are included in Appendix A of this report.

In this report we shall be primarily concerned with a series motor propulsion representation, a terrain description, which includes a power spectrum, and a primitive control law. All of these items required modifications to the digital program ROVER of reference 1. An updated input description to ROVER is presented in Appendix B. Appendix C contains a printout of a sample problem, which will subsequently be described.

SERIES MOTOR PROPULSION SYSTEM

The series motor model is based on a quasi-static approach suggested by I. Y. Bar-Itzhack of Bellcomm, Inc. The formulation can be stated as follows:

$$VOLT = (i)(RFIX + REV) + CONMEG(\omega)(i)^{a} \tag{1}$$

and

$$T_h = DCON(i)^{CMOT} , \tag{2}$$

where
VOLT = battery voltage
i = quasi-static current
a = CMOT-1
RFIX = fixed resistance
REV = variable resistance
ω = angular wheel velocity
CONMEG, DCON, CMOT = constants
T_h = theoretical torque.

The slip interaction model between the soil and the wheel adopted here is a polynomial formulation suggested to me by J. D. Richey of Bellcomm, Inc.

$$\omega = v/(RW)(1-S)$$

$$S = \sum_{i=0}^{n} a_i y^i \quad , \tag{3}$$

where y = T/(RW)(P), the pull number
RW = wheel radius
P = ground normal force for wheel f
T = torque (estimated from previous time step)
v = absolute value of forward wheel velocity
S = slip coefficient.

The instantaneous power (W) and torque (T) are given as follows

$$\begin{aligned} W &= (i)VOLT \\ T &= Th-TFR \end{aligned} \tag{4}$$

where TFR is the frictional torque associated with the rotating parts of the motor-wheel system. If a wheel is airborne the angular velocity at time step k is estimated from the torque at the previous time step (k-1) as follows

$$\omega_k = \omega_{k-1} + (\Delta_k)(T_{k-1})/I_W \tag{5}$$

where Δ_k is the time interval and I_W is the moment of inertia of the rotating parts.

The propulsion force for the wheel is obtained as follows:

$$F = (T)(C)/RW \tag{6}$$

where C is ratio of the sprung mass to total mass. F is limited in absolute value to TFxP where TF is an equivalent coefficient of friction for the soil.

TERRAIN DESCRIPTION

The elevation of the terrain is defined by a constant Gϕ, an X-slope Aϕ, a Y-slope Bϕ, a ramp slope CXϕ, $\cos^2$ bumps or craters, and a spatial realization of a random topography. Let us define the following terms:

$$S(CXA) = 1 \text{ for } (X - CXA) > 0$$
$$= 0 \text{ for } (X - CXA) < 0$$

$$\delta_\alpha = 1 \text{ if } |X - X_\alpha| < \frac{X_{h\alpha}}{2} \text{ and } |Y - Y_\alpha| < \frac{Y_{h\alpha}}{2} \quad ,$$

$$\delta_\alpha = 0 \text{ otherwise.}$$

The elevation $Z_o(X,Y)$ is given as follows:

$$Z_o(X,Y) = G\phi + A\phi(X-X\phi) + B\phi(Y-Y\phi) + S(CXA)\ CX\phi(X-CXA)$$
$$+ \sum_{\alpha=1}^{NG} \delta_\alpha H_\alpha \cos^2 \frac{\pi(X-X_\alpha)}{X_\alpha} \cos^2 \frac{\pi(Y-Y_\alpha)}{Y_{h\alpha}}$$
$$+ \sum_{\alpha=1}^{NCF} 2\left(AMC(\alpha)\right)^{1/2} \sin\left(2\pi(CFR(\alpha))\left(X - \frac{r_x(\alpha)}{CFR(\alpha)}\right)\right)$$
$$\sin\left(2\pi(CFR(\alpha))\left(Y - \frac{r_y(\alpha)}{CFR\ \alpha)}\right)\right) \quad . \tag{7}$$

For input (Appendix B):

$$HAL(\alpha) = H_\alpha$$
$$XAL(1,\alpha) = X_\alpha$$
$$XAL(2,\alpha) = Y_\alpha$$
$$XHAL(1,\alpha) = X_{h\alpha}$$
$$XHAL(2,\alpha) = Y_{h\alpha} \quad .$$

The last term in $Z_o(X,Y)$ is a spatial realization of a power spectrum where $CFR(\alpha)$ are center frequencies in cycles/amplitude, $AMC(\alpha)$ are associated powers in amplitudes squared, $r_x(\alpha)$ and $r_y(\alpha)$ are random numbers based on a uniform distribution between 0 and 1, which was suggested by S. N. Hou of Bellcomm, Inc.

A typical α-bump is shown in Figure 1, assuming NCF = 0 and CXϕ = 0.

CONTROL LAW

Let us first define a few terms.

VCR = tolerance limit on vertical velocity.
ACR = tolerance limit on vertical acceleration.
RVRA = resistance/time, variable resistance rate of series motors.
RVRB = resistance/time, variable resistance rate of series motors.
U(1) = forward velocity.
UAF = 1.25 |U(1)|.
UAG = .975 |U(1)|.
VGD = preferred forward velocity.
A1 = angle in degrees in interia $\overline{X}$-$\overline{Y}$ frame of reference from $\overline{X}$ to velocity component in X-Y.
A2 = angle in degrees in inertia X-Y frame of reference from $\overline{X}$ to destination point.
ANUV = A2-A1.
AST = maximum angle outside front wheel for Ackerman steering law.

We check first to ascertain if the destination point has been acquired. If yes, we are finished, if no, let us continue. If either the vertical velocity or vertical acceleration in absolute value exceeds the tolerance limits VCR or ACR, we throttle back by increasing the variable resistance of the series motors by the rate RVRA. If we are throttled back as far as possible, then braking commences until the condition ceases to exist. Suppose we are now at some point in time where neither the vertical velocity or the vertical acceleration has reached the tolerance limit. If VGD, our preferred forward velocity, is less than UAF, we throttle back by increasing the variable resistance by the rate RVRD. If we are throttled back as far as possible, then braking commences until this condition is rectified. Suppose now we are at some point in time where neither the vertical velocity or vertical acceleration has reached the tolerance limits, and further suppose that our preferred velocity VGD is greater than UAF. If VGD is less than UAG, we decrease the variable resistance by the rate RVRD. In either case, whether VGD is less or greater than UAF, we set the steering angle of the outside front wheel equal in absolute value to (.5) ANUV. The other wheel steering angles are set according to the ackerman steering law. The steering correction is not made if |U(1)|/VGD is less than .05 or if |ANUV| is less than 1 degree. In any event, allowance will be made to lock any wheel at zero steering angle and to make any series motor inoperative.

SAMPLE PROBLEM

The sample problem consists of the LRV being propelled by individual series motors (one per wheel) subjected to the primitive control system, and traversing a random terrain. The data used in this problem are estimates only but nevertheless approximate current properties of the vehicle. Some of the data was obtained from reference 2. Mr. H Reid of MSFC was kind enough to estimate the characteristics of the series motors, while Mr. J. D. Richey of Bellcomm, Inc. estimated slip characteristics of a typical soil which might be encountered. The statistical characteristics of the nominal topography is shown in Table 1 and was chosen to represent a rough mare topography. Nominal slip coefficients vs. pull numbers are given in Table 2.

Three runs were made on the Univac 1108 digital computer at Bellcomm, Inc. Run no. 1 consisted of the nominal topography (rough mare). In run no. 2 the power spectrum of the topography was reduced by 10 db, placing it well into the smooth mare range. Run no. 3 consisted of a perfectly smooth terrain.

The control system endeavored to maintain an 8 km/hr speed with tolerance limits set at 10 inches per second vertical velocity and 40 inches per second^2 vertical acceleration. The results of the three runs as related to energy consumption have been tabulated in Table 3. Since the input data used in these runs are estimates subject to verification, and the lengths of the runs were extremely short (less than 30 seconds elapsed time) the results should not be considered extremely accurate. Nevertheless, the qualitative nature of the results indicate a small increase ($\sim$3%) in energy consumption when going from a perfectly smooth terrain to the smooth mare and a significant increase in energy consumption ($\sim$30%) when going from the smooth mare to the rough mare.

Included in Appendix C is a printout of the input data and typical output display for run no. 2 plus a description of the output display.

2031-SK-JCT

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Attachments

REFERENCES

1. S. Kaufman, The Equations of Motion of the Lunar Roving Vehicle, TM-70-2031-1, Case 320, Bellcomm, Inc., March 31, 1970.

2. S. Kaufman, Trip Report - Meeting on LRV Stability Analysis at MSFC, June 5, 1970, Case 320, Bellcomm, Inc.

Center Frequency		Power Spectrum	Power	
Cycles/Meter	Cycles/Inch	M^2/Cycles/Meter	$Meters^2$	$Inches^2$
.01	.00025	3.0	.03	48
.02	.0005	2.0	.02	32
.03	.00075	1.0	.01	16
.04	.001	.80	.008	12.8
.05	.00125	.60	.006	9.6
.06	.0015	.40	.004	6.4
.07	.00175	.20	.003	4.8
.09	.00225	.10	.0025	4.0
.12	.003	.07	.00215	3.3
.16	.004	.035	.0016	2.6
.21	.00525	.01	.0006	.96
.28	.007	.007	.0006	.96
.38	.0095	.005	.00055	.90
.50	.0125	.004	.0005	.80
.63	.01575	.002	.0003	.48
.80	.020	.0015	.00028	.43
1.00	.025	.001	.0002	.32

TABLE 1

NOMINAL TOPOGRAPHY - ROUGH MARE

Pull No.*	Slip Coefficient
0.0	-.15
.2	.10
.4	.3
.6	.7
.8	.8
1.0	.95

*Pull No. = Torque/(Wheel radius)(Normal ground force)

For Pull Nos. in excess of 0.5 use slip coefficient evaluated at .5.

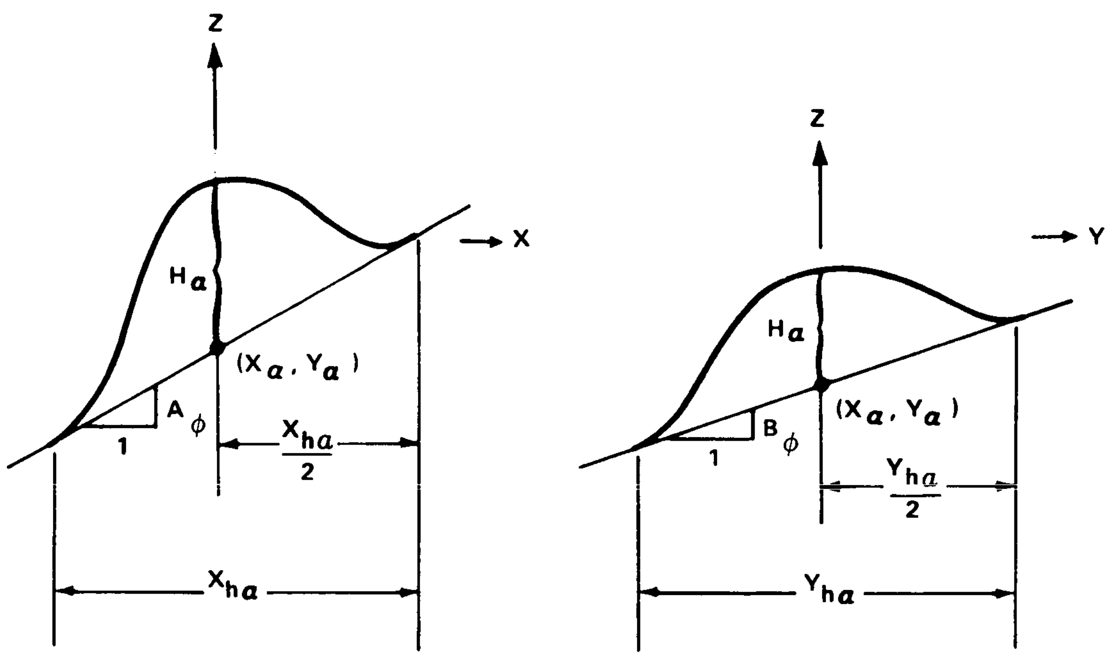
TABLE 2

NOMINAL SLIP COEFFICIENTS VERSUS PULL NUMBER

Run No	Elapsed Time of Run	Distance Traversed	Energy Dissipated	Energy Rate
	Seconds	Kilometers (KM)	Watt-Hrs	$\frac{\text{Watt-Hrs}}{\text{KM}}$
1. Rough Mare	19	.0407	4.67	114.7
2. Smooth Mare	19	.0420	3.65	86.5
3. Smooth Terrain	25	.0550	4.60	83.7

Table 3

POWER CONSUMPTION - SAMPLE PROBLEM



α -bump

H_α is minus for an α -crater

FIGURE 1 - α - BUMP GEOMETRY

APPENDIX A

DYNAMICS OF A FOUR-WHEEL VEHICLE

The model adopted contains six rigid body degrees-of-freedom and is subjected to gravity forces, ground-wheel interaction forces and non-linear inertia forces. The linear inertia accelerations in body coordinates are then solved in terms of forces described above. If a wheel becomes airborne the system simultaneously gains an inertia force and loses a ground-wheel force. The origin of the body coordinate system need not correspond to the center of gravity; any convenient origin will do.

The orientation of the vehicle with respect to a set of inertia coordinates (fixed on the moon) is shown in Fig. A1. Let $\bar{I}$, $\bar{J}$, $\bar{K}$ and $\bar{i}$, $\bar{j}$, $\bar{k}$ be unit vectors along the inertia (X,Y, Z) and body (x,y,z) coordinate system, respectively. The relationship between these vectors can be described in terms of a 3x3 direction cosine matrix [B]. This relationship in terms of the Euler angles is derived in reference A1 and is given below:

$$\begin{Bmatrix} \bar{I} \\ \bar{J} \\ \bar{K} \end{Bmatrix} = \begin{bmatrix} B \end{bmatrix} \begin{Bmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{Bmatrix} , \tag{A1}$$

where

$$\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\phi-\cos\theta\sin\phi\sin\psi & -\sin\psi\cos\phi-\cos\theta\sin\phi\cos\psi & \sin\theta\sin\phi \\ \cos\psi\sin\phi-\cos\theta\cos\phi\sin\psi & -\sin\psi\sin\phi+\cos\theta\cos\phi\cos\psi & -\sin\theta\cos\phi \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{bmatrix} .$$

If we differentiate Eq. (A1) with respect to time we obtain the following useful relationship:

$$[\dot{B}] = -[B][C] , \tag{A2}$$

where

$$[C] = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} .$$

The terrain elevation is a defined function $Z_o(X,Y)$ with derivatives $dZ_o/dX = \tan\gamma_x$ and $dZ_o/dY = \tan\gamma_y$.

Then

$$\sec\gamma_x = \left(1 + \tan^2\gamma_x\right)^{1/2} \quad ,$$

and

$$\sec\gamma_y = \left(1 + \tan^2\gamma_y\right)^{1/2} \quad . \tag{A3}$$

The unit tangent vectors to the terrain surface in the $\bar{I}$-$\bar{K}$ and $\bar{J}$-$\bar{K}$ planes are respectively:

$$\bar{t}_x = \cos\gamma_x\ \bar{I} + 0\bar{J} + \sin\gamma_x\ \bar{K} \quad ,$$

and

$$\bar{t}_y = 0\bar{I} + \cos\gamma_y\ \bar{J} + \sin\gamma_y\ \bar{K} \quad . \tag{A4}$$

The outward normal vector to the surface is next computed.

$$\bar{t}_n = \frac{\bar{t}_x \times \bar{t}_y}{|\bar{t}_x \times \bar{t}_y|} = \{a_n\}^t \begin{Bmatrix} \bar{I} \\ \bar{J} \\ \bar{K} \end{Bmatrix} \quad , \tag{A5}$$

where

$$\{a_n\}^t = \frac{1}{\ell_n} \left\lfloor -\sin\gamma_x\cos\gamma_y \quad -\sin\gamma_y\cos\gamma_x \quad \cos\gamma_x\cos\gamma_y \right\rfloor \quad ,$$

and

$$\ell_n = \left(\cos^2\gamma_x + \sin^2\gamma_x\cos^2\gamma\right)^{1/2} \quad .$$

In Fig. A2 is shown double Ackerman steering geometry for the vehicle. Double Ackerman is merely one possible steering mode, it is possible to allow the steering angles (a_f,f=1,2, 3,4) to be set independent of each other and possess their own steering rates. For wheel f the following relationship exists.

$$\begin{Bmatrix} \bar{i} \\ \bar{j} \\ \bar{k} \end{Bmatrix} = [AAA]_f \begin{Bmatrix} \bar{i}_f \\ \bar{j}_f \\ \bar{k} \end{Bmatrix} , \tag{A6}$$

where

$$[AAA]_f = \begin{bmatrix} \cos a_f & -\sin a_f & 0 \\ \sin a_f & \cos a_f & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

In terms of the inertia frame of reference, the wheel triad is given as follows:

$$\begin{Bmatrix} \bar{I} \\ \bar{J} \\ \bar{K} \end{Bmatrix} = [AAI]_f \begin{Bmatrix} \bar{i}_f \\ \bar{j}_f \\ \bar{k} \end{Bmatrix} , \tag{A7}$$

where

$$[AAI]_f = [B][AAA]_f .$$

The bump or crater dimensions may be of the same size as the wheel diameter. This situation creates a problem in finding a suitable wheel-spring stroke for calculating a ground-vehicle force normal to the wheel surface. The following rational approach has been adopted. With the wheels and suspension fully extended, the circumference of the wheel is searched for apparent locations imbedded beneath the ground surface. A unit vector $\bar{\tau}_1$ is constructed from the last point (b) to the first point (a) lying beneath the surface (Fig. A3). A slope s_1 is computed, a representative point (g) (mid-point) found with coordinates $\{XYZ\}_g$. Next, a unit vector $\bar{\tau}_2$, normal to both $\bar{\tau}_1$ and the theoretical ground normal $\bar{t}_{ng}$ (at $\{X_g Y_g\}$, Eq. A5) is found as follows:

$$\bar{\tau}_2 = \frac{\bar{t}_{ng} \times \bar{\tau}_1}{|\bar{t}_{ng} \times \bar{\tau}_1|} \quad . \tag{A8}$$

The slope along $\bar{\tau}_2$ is denoted s_2. An "average" ground normal $\bar{\tau}_3$ used in subsequent calculations is then computed,

$$\bar{\tau}_3 = \bar{\tau}_2 \times \bar{\tau}_1 \quad . \tag{A9}$$

The following relationship can be constructed from Eqs. (A8) and (A9):

$$\begin{Bmatrix} \bar{\tau}_1 \\ \bar{\tau}_2 \\ \bar{\tau}_3 \end{Bmatrix} = [G]_f \begin{Bmatrix} \bar{I} \\ \bar{J} \\ \bar{K} \end{Bmatrix} \quad . \tag{A10}$$

Now,

$$\begin{Bmatrix} \bar{\tau}_1 \\ \bar{\tau}_2 \\ \bar{\tau}_3 \end{Bmatrix} = [H]_f \begin{Bmatrix} \bar{i}_f \\ \bar{j}_f \\ \bar{k} \end{Bmatrix} \tag{A11}$$

where

$$[H]_f = [G]_f [B] \quad .$$

Point (e) as shown in Fig. A3 is next obtained and an apparent wheel penetration along $\bar{\tau}_3$ is computed from point h to point (e) and is denoted $\Delta\tau_f$.

Next, the body coordinates of point (e) are found relative to the origin of the vehicle. These coordinates will be denoted $\{xyz\}_e$. Let $\{uvw\}$ denote the velocity of the vehicle's origin along the body axes. The velocity of the wheel f at point (e) in the inertia frame of reference is given as follows,

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_f = [B]\left(\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} - [C]\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_e\right) \quad . \tag{A12}$$

The velocity along the $\{\bar{\tau}_1\bar{\tau}_2\bar{\tau}_3\}$ triad is next obtained from Eqs. (A10) and (A12).

We shall now evaluate the wheel-ground force in the direction $\bar{\tau}_{3f}$. Let spring constants along the triad $\{\bar{i}_f\ \bar{j}_f\ \bar{k}\}$ be denoted as $\{s_{if}\ s_{jf}\ s_{kf}\}$, spring lengths as $\{\ell_{if}\ \ell_{jf}\ \ell_{kf}\}$, damping constants as $\{0\ 0\ c_{kf}\}$, spring deflections as $\{\Delta u_{if}\ \Delta u_{jf}\ \Delta u_{kf}\}$, and forces as $\{P_{if}\ P_{jf}\ P_{kf}\}$. Forces along the triad $\{\bar{\tau}_1\ \bar{\tau}_2\ \bar{\tau}_3\}$ will be denoted $\{P_{1f}\ P_{2f}\ P_{3f}\}$. The springs will be considered non-linear, that is,

$$s_{\alpha f} = s_{\alpha af} (\alpha = i,j,k) \qquad \text{if} \qquad \ell_{\alpha f} > |\Delta u_{\alpha f}| \quad ,$$

$$s_{\alpha f} = s_{\alpha bf} \qquad \text{if} \qquad \ell_{\alpha f} < |\Delta u_{\alpha f}| \quad ,$$

and

$$s_{\alpha bf} >> s_{\alpha af} \quad .$$

The three spring-deflection relationships are:

$$[I]\begin{Bmatrix} P_{if} \\ P_{jf} \\ P_{kf} \end{Bmatrix} - \begin{bmatrix} s_{if} & & \\ & s_{jf} & \\ & & s_{kf} \end{bmatrix}\begin{Bmatrix} \Delta u_{if} \\ \Delta u_{jf} \\ \Delta u_{kf} \end{Bmatrix} = - \begin{bmatrix} \delta_{if}(s_{ibf}-s_{iaf})\ell_{if} \\ \delta_{jf}(s_{jbf}-s_{jaf})\ell_{jf} \\ \delta_{kf}(s_{kbf}-s_{kaf})\ell_{kf} \end{bmatrix}$$

$$- \begin{bmatrix} 0 \\ 0 \\ H_f(3,3)v_{\tau_{3f}} c_{kf}\ |v_{\tau_3 f}|^{n-1} \end{bmatrix} \quad , \tag{A13}$$

where

$$n = 1 \text{ for linear damping}$$

$$\delta_{\alpha f} = 1 \quad \text{if} \quad \ell_{\alpha f} < |\Delta u_{\alpha f}|$$

and

$$\delta_{\alpha f} = 0 \qquad \text{if} \qquad \ell_{\alpha f} > |\Delta u_{\alpha f}| \quad .$$

Equilibrium of tangential forces requires

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \end{bmatrix} \begin{Bmatrix} P_{if} \\ P_{jf} \\ P_{kf} \end{Bmatrix} = \begin{Bmatrix} P_{1f} \\ P_{2f} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad . \tag{A14}$$

Deflection compatability requires

$$\lfloor H_{31} \; H_{32} \; H_{33} \rfloor \begin{Bmatrix} \Delta u_{if} \\ \Delta u_{jf} \\ \Delta u_{kf} \end{Bmatrix} = \Delta_{\tau f} \quad . \tag{A15}$$

Equations (A13, A14 and A15) are combined into one matrix equation as follows:

$$[E1]\{T2\} = \{T1\} \tag{A16}$$

where

$$\{T1\} = \begin{Bmatrix} -\delta_{if}(s_{ibf}-s_{iaf})\ell_{if} \\ -\delta_{jf}(s_{jbf}-s_{jaf})\ell_{jf} \\ -\delta_{kf}(s_{kbf}-s_{kaf})\ell_{kf} - H_f(3,3)v_{\tau_{3f}}c_{kf}\ |v_{\tau_3 f}|^{n-1} \\ 0 \\ 0 \\ \Delta_{\tau f} \end{Bmatrix}$$

$$\{T2\} = \{\Delta u_{if}\ \Delta u_{jf}\ \Delta u_{kf}\ P_{if}\ P_{jf}\ P_{kf}\} \quad ,$$

and

$$[E1] = \begin{bmatrix} -s_{if} & 0 & 0 & 1 & 0 & 0 \\ 0 & -s_{jf} & 0 & 0 & 1 & 0 \\ 0 & 0 & -s_{kf} & 0 & 0 & 1 \\ 0 & 0 & 0 & H_{11} & H_{12} & H_{13} \\ 0 & 0 & 0 & H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} & 0 & 0 & 0 \end{bmatrix} \quad .$$

After solving Eq. A16, the normal ground force is obtained as follows,

$$P_{3f} = H_{31}\, P_{if} + H_{32}\, P_{jf} + H_{33}\, P_{kf} \quad . \qquad \text{(A17)}$$

We will now compute P_{2f} along $\bar{\tau}_{2f}$. Let VSE be some small velocity and M_s an equivalent coefficient of sidewise friction. Then,

$$P_{2f} = \frac{-v_{\tau 2}}{VSE} M_s P_{3f} \qquad \text{for} \qquad |\bar{v}_{\tau 2}| < VSE \quad , \qquad \text{(A18)}$$

and

$$P_{2f} = \frac{-v_{\tau 2}}{|v_{\tau 2}|} M_s P_{3f} \qquad \text{for} \qquad |\bar{v}_{\tau 2}| > VSE \quad . \qquad \text{(A19)}$$

Forces in the $\bar{\tau}_{1f}$ direction can come about thru rolling friction, braking friction, or engine torque forces. These forces for each wheel will be denoted P_{1f}.

The wheel forces rotated in $\bar{i}$, $\bar{j}$, $\bar{k}$ directions are

$$\begin{Bmatrix} P_i \\ P_j \\ P_k \end{Bmatrix}_f = [B]^t [G]_f^t \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix}_f \quad . \tag{A20}$$

The summation of these forces and moments about the origin of the body coordinate system are as follows:

$$\{P_w\} = [D_f] \begin{Bmatrix} P_i \\ P_j \\ P_k \end{Bmatrix}_f \tag{A21}$$

where

$$[D_f] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -z_e & y_e \\ z_e & 0 & -x_e \\ -y_e & x_e & 0 \end{bmatrix} \quad .$$

For all four wheels,

$$\{P_w\} = \sum_{f=1}^{4} \{P_w\}_f \quad . \tag{A22}$$

The mass matrix [M] relative to the body coordinate system is defined as follows:

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & H_z & -H_y \\ & m & 0 & -H_z & 0 & H_x \\ & & m & H_y & -H_x & 0 \\ & & & I_{xx} & -I_{xy} & -I_{xz} \\ & \text{symmetrical} & & & I_{yy} & -I_{yz} \\ & & & & & I_{zz} \end{bmatrix} , \qquad \text{(A23)}$$

where m is the sprung mass, and

$$H_x = \int x dm \quad , \quad H_y = \int y dm \quad , \quad H_z = \int z dm$$

$$I_{xy} = \int xy dm \quad , \quad I_{xz} = \int xz dm \quad , \quad I_{yz} = \int yz dm$$

$$I_{xx} = \int (y^2 + z^2) dm \quad , \quad I_{yy} = \int (x^2 + z^2) dm \quad , \quad I_{zz} = \int (x^2 + y^2) dm .$$

The three linear and three angular momentum components {L} along the body coordinate system is given by the following relationship.

$$\{L\} = [M]\{U\} \qquad \text{(A24)}$$

where,

$$\{U\} = \{u \ v \ w \ \omega_x \ \omega_y \ \omega_z\} .$$

The forces and moment components along the body coordinate axes are given as follows:

$$\{\dot{L}\} = [M]\{\dot{U}\} - \{P_I\} \qquad \text{(A25)}$$

where

$$\{P_I\} = [C_1][M]\{U\} \quad ,$$

and

$$[C_1] = \left[\begin{array}{c|c} [C] & 0 \\ \hline 0 & [C] \end{array}\right] \quad .$$

Gravity forces act along the negative $\bar{K}$ axis thru the cg. Gravity force and moment components about the body origin are given below:

$$\{P_g\} = [M]\{-B_{31}g_m \;\; -B_{32}g_m \;\; -B_{33}g_m \;\; 0 \;\; 0 \;\; 0\} \quad , \tag{A26}$$

where g_m is the moon's acceleration of gravity.

The equations of motion of the system can now be stated as follows (see Eqs. (A22, A25, A26)):

$$\{\dot{U}\} = [M]^{-1}\{P\} \tag{A27}$$

where

$$\{P\} = \{P_g\} + \{P_w\} + \{P_I\}$$

The velocities along the body axes are updated from time t=i to t=i+h as follows:

$$\{U\}_{i+h} = \{U\}_i + h\{\dot{U}\}_i \quad . \tag{A28}$$

Let

$$\begin{aligned} \theta_j &= (\omega_j(i+h) + \omega_j(i))(.5)(h), \quad j = x,y,z \quad , \\ \theta &= (\theta_x{}^2 + \theta_y{}^2 + \theta_z{}^2)^{1/2} \quad . \end{aligned} \tag{A29}$$

Given the direction cosine matrix [B] at t = i, it can be updated as follows (reference A2):

$$[B]_{i+h} = [B]_i \left([I] - \frac{\sin\theta[\phi]}{\theta} + \frac{(1 - \cos\theta)[\phi]^2}{\theta^2} \right) , \qquad (A30)$$

where $[\phi]$ has the same form as [C] (θ_j replaces ω_j in Eq. (A2)).

The algorithm for obtaining equation (A30) was obtained from I. Y. Bar-Itzhack of Bellcomm, Inc., and is known as the subroutine DICOS.

The trajectory is updated as follows:

$$\begin{Bmatrix} X_B \\ Y_B \\ Z_B \end{Bmatrix}_{i+h} = \begin{Bmatrix} X_B \\ Y_B \\ Z_B \end{Bmatrix}_i + {}^{h}/_{2} \left([B]_{i+h} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}_{i+h} + [B]_i \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}_i \right) , \qquad (A31)$$

where $\{X_B \ Y_B \ Z_B\}$ are the inertia coordinates of the origin of the vehicle.

The time interval h is tentatively set at DELTIM, an input variable. The velocity of each wheel (point(e)in Fig. A3) normal to the ground is continuously monitored. A maximum velocity in absolute value (ER) of the four wheels is obtained and a characteristic time TET = $(.05)(RW)/_{ER}$ is obtained (RW is the wheel radius). The number of intervals (L5) of the maximum interval (DELTIM) is computed in integer arithmetic as follows:

$$L5 = DELTIM/_{TET} + 1 \quad . \qquad (A32)$$

The integration algorithm is based on the well known Runge-Kutta method and will not be discussed here.

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REFERENCES

A1. Goldstein, H., Classical Mechanics, Addison-Wesley, Chapter IV, 1953.

A2. Wilcox, J. C., A New Algorithm for Strapped-Down Inertial Navigation, IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-3, No. 5, September 1967.

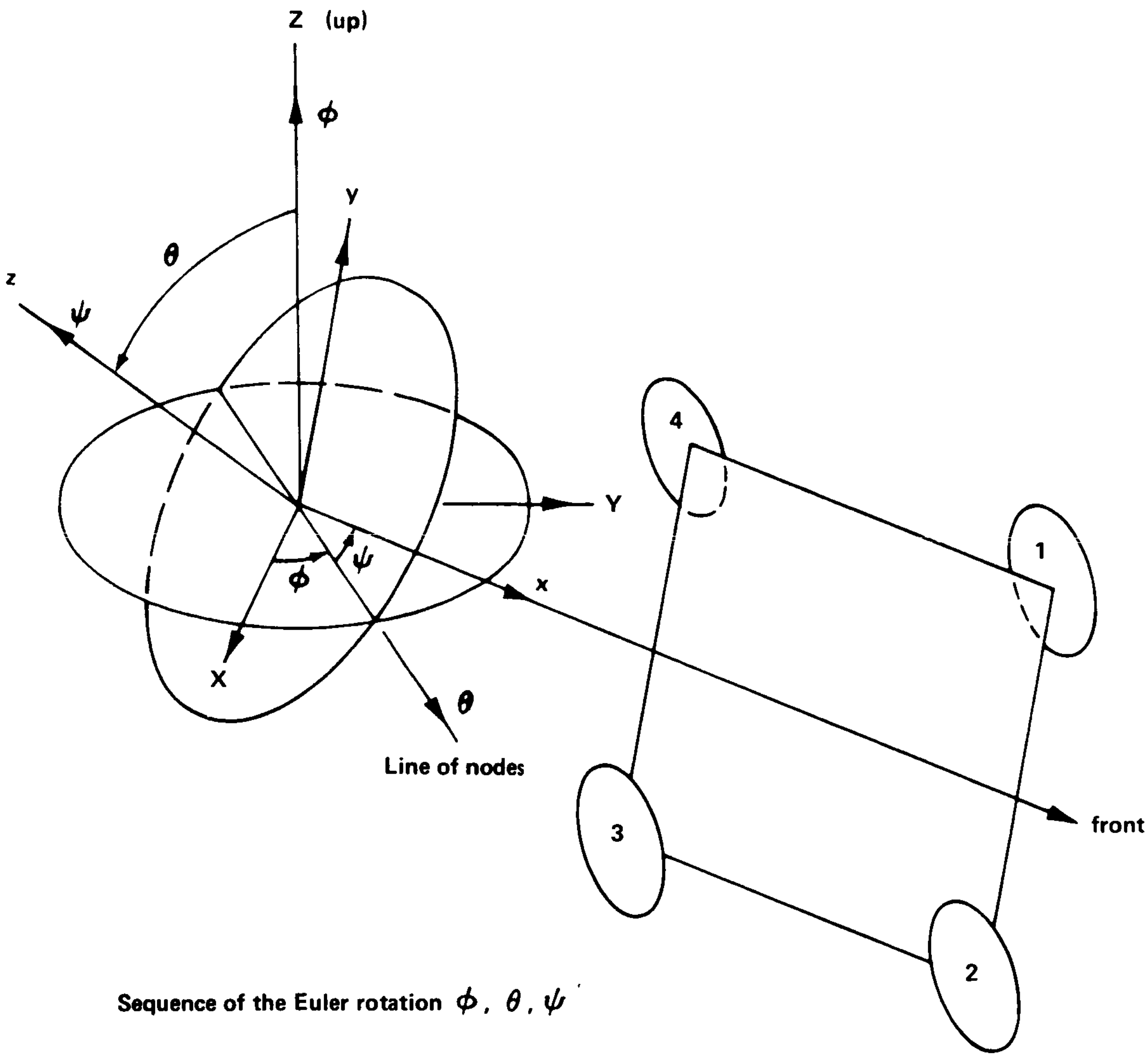
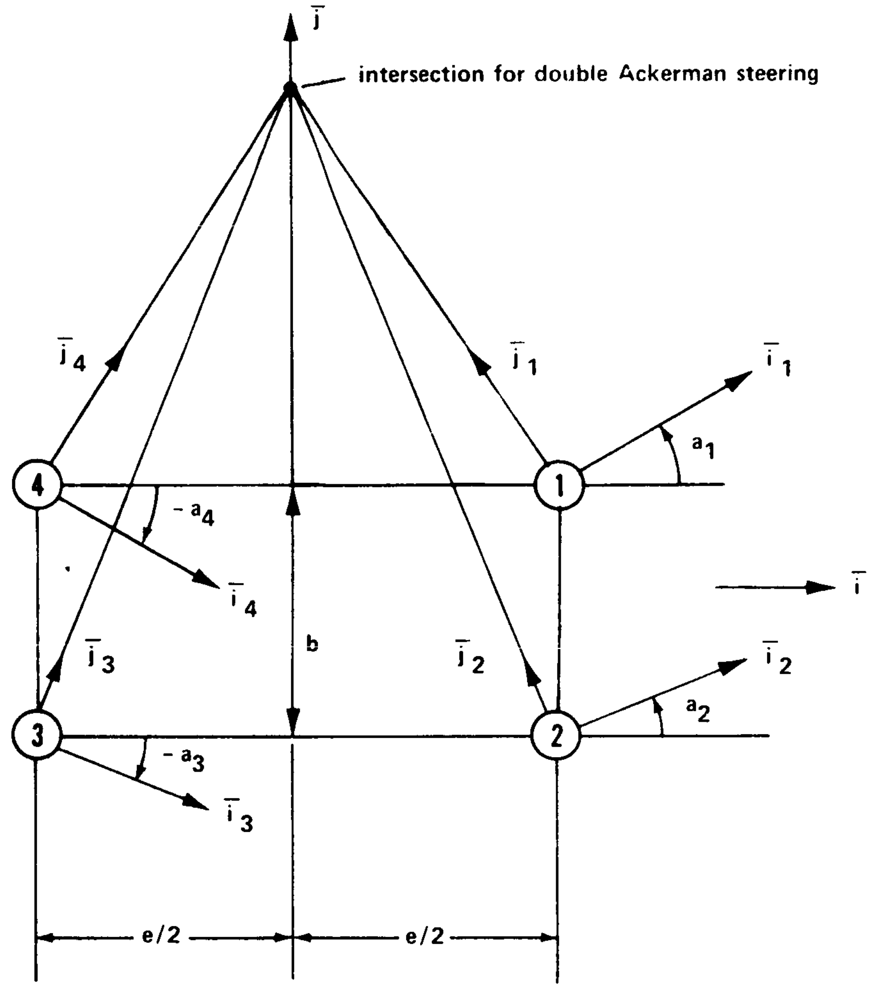


FIGURE A1 - AXIS ORIENTATION

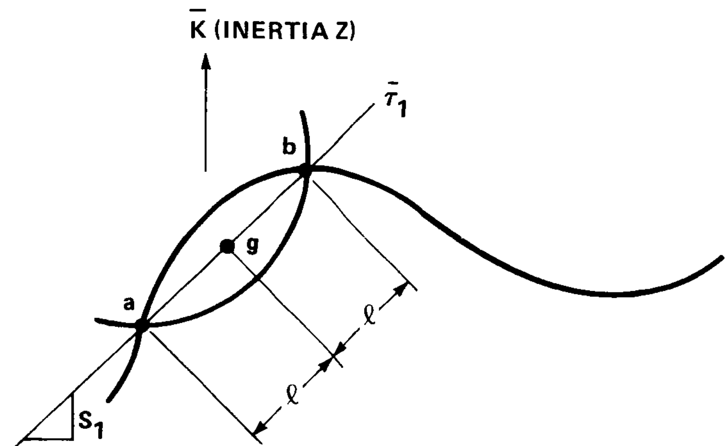


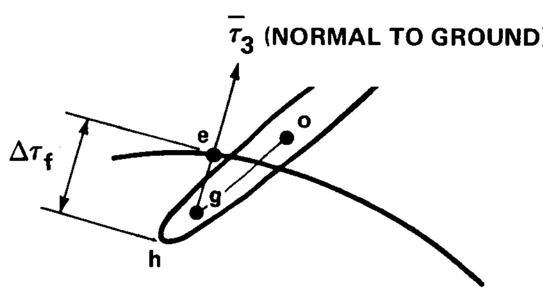
For double Ackerman steering let β be steering angle - front outside wheel.

$\beta < 0 \quad a_1 = -a_4 = \beta \qquad a_2 = -a_3 = \tan^{-1}\left(e/2 \tan\beta / (e/2 + b\tan\beta)\right)$

$\beta > 0 \quad a_2 = -a_3 = \beta \qquad a_1 = -a_4 = \tan^{-1}\left(e/2 \tan\beta / (e/2 - b\tan\beta)\right)$

FIGURE A2 - STEERING GEOMETRY





POINT g LIES ON WHEEL HALFWAY BETWEEN a AND b
POINT e LIES ON SURFACE ALONG $\bar{\tau}_3$ THRU g
POINT o IS CENTER OF HUB
POINT h LIES ON CIRCUMFERENCE OF WHEEL
PROJECTED FROM o-g

FIGURE A3 - WHEEL GROUND INTERACTION WHEEL FULLY EXTENDED

APPENDIX B

INPUT TO ROVER

The digital program ROVER is continually updated. The current input for the UNIVAC 1108 is in NAMELIST ($NAMI) format and is given below.

General Input

X = single array of order 3 of the inertia X,Y,Z coordinates of the origin of the vehicle.

Z = double array of order 3x4 containing the body x,y,z coordinates of the wheel hubs (wheel and suspension fully extended).

RW = wheel radius.

PHI = initial ϕ angle in degrees (Fig. 1).

THETA = initial θ angle in degrees (Fig. 1).

PSI = initial ψ angle in degrees (Fig. 1).

U = single array of order 6 containing initial rates $\{u\ v\ w\ \omega_x\ \omega_y\ \omega_z\}$.

NINT = number of time intervals (DELTIM) for integration.

DELTIM = time span of one time interval (maximum).

IPRT = printing integer (print every IPRT time interval).

NDOT = number of locations for additional acceleration output (not to exceed 20).

R = double array (3)(NDOT) of body x,y,z coordinates.

GM = moon's acceleration of gravity.

Weight Data

CM = constant to divide all weight data (mass conversion constant) CM = 0 implies CM = 1.

WM = weight of one unsprung wheel mass; in what follows do not include the unsprung wheel weight.

NMASS = 0 implies zero, first, and second weight moments will be supplied.

Y(1) = m (total sprung weight rather than sprung mass).

Y(2) = I_{xx}.

Y(3) = I_{yy}.

Y(4) = I_{zz}.

Y(5) = H_x.

Y(6) = H_y.

Y(7) = H_z.

Y(8) = I_{xy}.

Y(9) = I_{xz}.

Y(10) = I_{yz}.

NMASS = 1 implies detail weight breakdown.

NIT = number of masses, not to exceed 30.

A(J,α) = double array of order (NIT)(7) of weight data.

α = 1, weight.

α = 2, x coordinate.

α = 3, y coordinate.

α = 4, z coordinate.

α = 5, local x moment of inertia (about own cg).

α = 6, local y moment of inertia.

α = 7, local z moment of inertia DIMENSION A(30,7).

Elevation Data

NXY = 0, implies CXϕ = 0.

NXY = 1, implies CXϕ may have a non-zero value.

CXA = X, location.

CXϕ = slope parameter.

Aϕ = X-slope.

Bϕ = Y-slope.

Gϕ = constant amplitude.

Xϕ = origin for definition of X-slope.

Yϕ = origin for definition of Y-slope.

NG = number of α-bumps/craters.

XAL(β,α) = double array of order 2xNG to locate α-bumps/crater,
β = 1 is X-coordinate (X_α),
β = 2 is Y-coordinate (Y_α).

XHAL(β,α) = double array of order 2xNG for α-bump/crater diameters,
β = 1 is $X_{h\alpha}$,
β = 2 is $Y_{h\alpha}$.

HAL(α) = amplitudes (H_α).

NCF = number of center frequencies for random terrain, not to exceed 25.

CFR = single array of order NCF of center frequencies (cycles per amplitude).

AMC = single array of order NCF of power associated with center frequencies (amplitude squared).

Friction Force Data

Let AV1, AV2 be absolute value of wheel velocity along $\overline{\tau 1}$ and $\overline{\tau 2}$, respectively.

SF = coefficient of sidewise friction.
SF = 0 implies frictionless surface along $\overline{\tau 2}$.

VSE = minimum velocity for full sidewise friction.
If AV2 < VSE then sf(equivalent) = SF x AV2/VSE.

NRØL(f) = single array of order 4.
0 implies no rolling friction for wheel f.
1 implies rolling friction for wheel f.
If NGUID = 1 or NSTER = 1, NROL(f) is automated into the applicable algorithm. If NBRAK(f) = 1 then NROL(f) is assumed zero.

RF = coefficient of rolling friction.
RF = 0 implies no rolling friction.

VROL = minimum velocity for full rolling friction.
If AV1 < VROL, then rf(equivalent) = RF x AV1/VROL.

NBRAK(f) = single array of order 4.
0 implies no braking for wheel f.
1 implies braking for wheel f.
If NGUID = 1 or NSTBR = 1, NBRAK(f) is automated into the applicable algorithm.

BF = coefficient of braking friction, zero implies braking is inoperative.

VBRAK = minimum velocity for full braking friction.
If AV1 < VBRAK, then bf(equivalent) = BF x AV1/VBRAK.

CONS(f) = single array of order 4. Zero implies braking is inoperative for wheel f. The braking force is a minimum of CONS(f) and BF PNF(3,f) where PNF(3,f) is the normal ground force (along $\overline{\tau 3}$).

NTORQ(f) = single array of order 4.
0 implies no engine torque for wheel f.
1 implies engine torque for wheel f.

TF = coefficient of ground friction torque for wheel f. TF=0 implies no engine torque.

UNIT = $\pm$ 1.0, 1.0 implies forward motion, -1.0 implies rearward motion, and 0 implies no engine torque.
If NSERMO = 1 then NTORQ(f) is tentatively set equal to 1 and the remaining velocity-torque data now described has a different meaning. See NSERMO = 1.

NP = number of pieces of data (not to exceed 6) from which to construct a torque-velocity polynomial of order NP-1.

VEL = single array of velocities of order NP.

TORQ = single array of torques of order NP.

VMAX = velocity limit above which torque = 0.

VMIN = velocity limit below which torque equals its polynomial value at VMIN.

<u>Suspension Characteristics (see Eq. A13)</u>

N50 = 0 implies maximum input formation.

SL(α,f) = spring lengths for wheel f $\{\ell_{if}\ \ell_{jf}\ \ell_{kf}\}$.

SIA(α,f) = soft spring constants for wheel f $\{s_{iaf}\ s_{jaf}\ s_{kaf}\}$.

SIB(α,f) = hard spring constants for wheel f $\{s_{ibf}\ s_{jbf}\ s_{kbf}\}$.

DAMP(f) = damping constant for wheel f DIMENSION SL(3,4), SIA(3,4), SIB(3,4), CS(3,4), DAMP(4).

DAMC = power of velocity dependent damping force (zero implies linear damping, DAMC = 1).

COLUMB = coulomb damping force for each wheel.

VCOUL = minimum vertical velocity to attain full calculated damping force, otherwise damping force = calculated force x $|V_{\tau_3 f}|$/VCOUL

N50 = 1 implies minimum input formation.

S(f) = ℓ_{kf}.

SA(f) = s_{kaf}.

SB(f) = s_{kbf}.

DAMP(f), = DAMC, COLUMB, VCOUL

N50 = 1 also implies the following;
$\ell_{if} = \ell_{jf} = 10\ell_{kf} = 10s(f)$,
$s_{iaf} = s_{jaf} = s_{ibf} = s_{jbf} = s_{kbf} = SB(f)$,
DIMENSION S(4), SA(4), SB(4), DAMP(4).

<u>Steering (see Fig. A2)</u>

AF(f) = constant steering angle for wheel f in degrees.

N100 = 1 implies time dependent constant rate Ackerman steering (see Fig. A2). Not applicable if NGUID = 1 or NSTBR = 1.
NSGAK = 0 implies double Ackerman,
NSGAK = 1 implies single Ackerman.

AG = initial steering angle in degrees for outside front wheel.

ST1 = steering rate in degrees/time for outside front wheel.

TIMA = initial time for onset of ST1.

TIMB = final time for ST1.

ST3 = steering rate in degrees/time for outside front wheel.

TIMC>TIMB = initial time for onset of ST3.

TIMD = final time for ST3; for N100=1 the steering angle, (AP), outside front wheel is given as follows:

AP1 = AG for t < TIMA,

AP2 = AG + ST1(t-TIMA) for TIMA < t < TIMB,

AP3 = AG + ST1(TIMA-TIMB) for TIMB < t < TIMC,

AP4 = AP3 + ST3(t-TIMC) for TIMC < t < TIMD,

AP5 = AP3 + ST3(TIMD-TIMC) for t > TIMD.

AST is a maximum outside wheel steering in degrees; AP1....AP5 in absolute value will be limited by AST.

NSIN = 1 implies sinusoidal Ackerman steering. Not applicable if NGUID = 1, NSTBR = 1, N100 = 1. NSGAK = 0 implies double Ackerman, NSGAK = 1 implies single Ackerman.

AP = (AMP)SIN(ST1)(t-Tϕ) for front outside wheel for 0 < t < TSIN.

AMP = amplitude in degrees (positive real only let ST1 take on desired sign).

Tϕ = time lag.

TSIN= final time.

NTRIM = 1 implies elevation will be automated to trim the vehicle for 1 lunar g. The algorithm goes as follows. In body coordinates construct a vector from the vehicle origin to the ground at the intersection of the wheel centers. Call this vector {ALm}, and:

ALX = (Z(1,1) + Z(1,4))/2.

ALY = (Z(2,1) + Z(2,2))/2.

ALZ = -RW + Z(3,1) + DH,
where DH = (mass)(GM)B(3,3)/(4)SIA(3,1).

Of course this vector assumes elastic and geometry symmetry.

This new origin in inertia coordinates is computed as follows.

$$\begin{Bmatrix} X\phi \\ Y\phi \\ G\phi \end{Bmatrix} = \begin{Bmatrix} X(1) \\ X(2) \\ X(3) \end{Bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{Bmatrix} ALX \\ ALY \\ ALZ \end{Bmatrix}$$

We must now compute slopes from a modified [B] matrix to account for the cg of the vehicle not coinciding with $X\phi$, $Y\phi$. Call this cg shift and corresponding to X&Y, respectively. This shift causes small angle changes THET1 and THET2, computed as follows:

$$\text{THET1} = (\text{DH})(\text{OFF2})/(\text{ALY}-\text{Z}(2,1))^2,$$

$$\text{THET2} = (-\text{DH})(\text{OFF1})/(\text{ALX}-\text{Z}(1,1))^2.$$

The [B] matrix can now be modified and a new direction cosine matrix [B1] computed. The desired slopes are now obtained as follows:

$$A\phi = -\text{B1}(1,3)/\text{B1}(3,3),$$

$$B\phi = -\text{B1}(2,3)/\text{B1}(3,3).$$

Ground vectors in terms of body vectors are given through the matrix $[BB] = [B1]^t[B]$. $v_{\tau 3} = 0$ implying $U(3) = -(BB(3,1)(U(1)) + BB(3,2)(U(2))/BB(3,3)$.

If NCF > 0 and NTRIM = 1, then AO, BO, and U(3) are set equal to zero and,

$$G\phi = G\phi \text{ (previously computed)} - .25\ (Z_1 + Z_2 + Z_3 + Z_4).$$

Z_f is the contribution to the elevation under wheel f of the random terrain at time = 0.

Series Motor

NSERMO = 1 implies series motor.

MST(f) = array of order 4.
1 implies operative motor for wheel f.
0 implies inoperative motor for wheel f.

See NTORQ(f) for comparisons to the following.

NP = number of pieces of data (not to exceed 6) from which to construct a Slip-X polynomial, (X = TORQUE/(RW)(NORMAL GRD. FORCE), slip numbers).

The polynomial is of order NP-1.

VEL = single array X's of order NP.

TORQ = single array of SLIP values of order NP, the data is of course a function of the soil.

VMAX = X limit above which Slip equals its polynomial value at VMAX.

REMAX = maximum variable resistance.

RFIX = fixed resistance.

COMMEG = back emf constant.

VOLT = battery voltage.

DCON = torque constant.

CMOT = exponent to raise current to obtain theoretical torque.

TFR = frictional torque. This is in addition to soil rolling torque.

TIN = moment of inertia of rotating motor-wheel system. If not given TIN = .5(WM)*RW^2. TIN will also be divided by CM.

Control System

NGUID = 1 implies control system.

XGD = X - inertia coordinate of destination.

YGD = Y - inertia coordinate of destination.

VCR = velocity tolerance limit.

ACR = acceleration tolerance limit. If either VCR or ACR is exceeded variable resistance of series motors, and/or braking of the wheels is affected.

VGD = preferred forward velocity of vehicle.

RVRA = resistance per time; rate of increase of variable resistance of series motor if VCR or ACR is exceeded.

LP = number of points to construct the rate RVRD (see Control Law) for the variable resistance.

CUC = single array of order LP of the operating currents.

RVRB = single array of order LP of rates.

ROP = a value of variable resistance.

Rev (variable resistance) = rev±(RVRD)(time). The arrays CUC and RVRB are used to construct a polynominal of order LP-1 for rate (RVRD, see Control Law) vs current. If in the process of decreasing the variable resistance (REV) the average current in the four motors remains below CUC(1) the variable resistance is set equal to ROP. Hence ROP must be chosen small enough so that the current always lies above CUC(1) but not so small that the resulting current is much larger than that required for the breaking torque (TFR). A good value for CUC(1) is somewhat less than that required for braking torque. On the other hand, if in the process of changing the variable resistance the current exceeds CUC(LP) it will be set equal to CUC(LP). Therefore CUC(LP) should be set equal to the maximum possible current (assuming REV and angular velocity vanish).

AST = maximum angle, in degrees, of outside front wheel for Ackerman steering law.

NSGAK = 0 implies double Ackerman.
1 implies single Ackerman.

POWIN = overhead power consumption. Does not include motor-controller circuit; includes for instance, navigation system, displays, steering power requirements, etc.

NST(f) = array of order 4.
1 implies steering operative for wheel f.
0 implies steering angle set equal to zero at all times for wheel f.

Braking and Steering Rate Option

Cannot be used in conjunction with control system (NGUID = 1).

NSTBR = 1 implies steering and/or braking rate option.

Steering angles are in degrees.

$$0<t<\text{SA1(f)}:\text{angle(f)} = \text{AF(f)} + \text{SATE1(f)xt}$$

$$\text{SA1(f)}<t<\text{SA1(f)} + \text{SSEC(f)}:\text{angle(f)} = \text{AF(f)} + \text{SATE1(f)xSA1(f)} + \text{SATE2(f)xt}$$

$$\text{SA1(f)} + \text{SSEC(f)}<t:\text{angle(f)} = \text{AF(f)} + \text{SATE1(f)xSA1(f)} + \text{SATE2(f)xSSEC(f)}$$

AF(f) = initial steering angle for wheel f.

SATE1(f), SATE2(f) = steering rates for wheel f.

SA1(f), SSEC(f) = times for wheel f.

Breaking forces are limited by friction - see description of friction force data.

$$0<t<\text{BA1(f)}:\text{force(f)} = \text{CONS(f)} + \text{BATE1(f)xt}$$

$$\text{BA1(f)}<t<\text{BA1(f)} + \text{BSEC(f)}:\text{force(f)} = \text{CONS(f)} + \text{BATE1(f)xBA1(f)} + \text{BATE2xt}$$

$$\text{BA1(f)} + \text{BSEC(f)}<t:\text{force(f)} = \text{CONS(f)} + \text{BATE1(f)xBA1(f)} + \text{BATE2(f)xBSEC(f)}$$

CONS(f) = initial breaking force for wheel f.

BATE1(f), BATE2(f) = braking rates for wheel f.

BA1(f), BSEC(f) = times for wheel f.

APPENDIX C

SAMPLE PROBLEM PRINTOUT

Included in this appendix is a printout of the input data and the output after 19 seconds into the traverse for run no. 2 (smooth mare) described in the main section of this report.

The input data is in NAMELIST format ($NAMI), as described in Appendix B. The maximum integration interval DELTIM was taken as .1 second. The subintervals (integration interval) printed out with each time of the traverse is computed according to equation A32. For the rough mare case the program objected continually to this maximum integration interval and accordingly reduced it.

It might be instructive here to describe the output displayed at each second of the traverse. The body translational and angular velocities are contained in the {U} vector (eq. A24). The ratio of sidewise to forward velocity is U(2)/U(1). The instantaneous heading angle tangent is defined as B(2,1)/B(1,1), where [B] is the direction cosine matrix defined by eq. (A1). The X,Y,Z trajectory is the inertia coordinates of the body origin (eq. A31). The total acceleration along the body reference axes is $\{U\}_6 - ([C]\{U\}_3)$, where $([C]\{U\}_3)$ is a correction to the three translational rigid body accelerations ([C] is defined by eq. (A2)). Wheel indicator 0/1 implies on/off the ground, respectively. Total energy (watt-seconds for this problem) was computed from the power of the motors (eq. 5) plus an estimated power overhead of 60 watts. The polar vector is $(X_B^2 + Y_B^2)^{1/2}$, and the tangent of the polar angle is Y_B/X_B, where X_B and Y_B are trajectory inertia coordinates (eq. A31). The wheel-ground forces have been rotated into the body axes. The steering angles are defined in Figure A2.

The acceleration extra points are at the locations of the NDOT points defined in the input. The nonlinear inertia forces, the gravity forces, and the wheel forces are the vectors $\{P_I\}$ (eq. A25), $\{P_g\}$ (eq. A26) and $\{P_W\}$ (eq. A22), respectively. The body hub coordinates under deformation are the hub coordinates for the wheels fully extended plus the deformations $\Delta\tau_f$, shown in Figure A3. The angle displayed with the above coordinates is measured in the wheel +y direction from top of wheel (+z body) to point g in Figure A3. If the wheel is off the ground, point g is taken as the lowest point on the rim of the wheel (inertia-Z).

SAMPLE PROBLEM NO 2

```
@XQT            SK3190.ABS
 RA=16.,WM=25.0,CM=386.0,GM=61.8,Y=1270.0,522428.,1297052.,1388397.,
 NINT=10000,DELTIM=.10,IPRT=10,
 SF=.6,NROL=4*1,RF=.05,VSE=1.0,
 NSO=0,DAMC=2.0,DAMP=4*.1528,
 SL=10.,3.,13.5,10.,3.,13.5,10.,3.,13.5,10.,3.,13.5,
 SIA=600.,30.,9.4,600.,30.,9.4,600.,30.,9.4,600.,30.,9.4,
 SIB=600.,450.,450.,600.,450.,450.,600.,450.,450.,600.,450.,450.,
 Z=46.7,35.95,-24.63,46.7,-35.95,-24.63,-43.3,-35.95,-24.63,-43.3,35.95,-24.63,
 PHI=90.0,PSI=-90.0,THETA=-.26,
 NDOT=3,R=0.,0.,0.,-7.3,14.5,0.,-7.3,-14.5,0.,
 VBRAK=1.,NTRIM=1,
 CONMEG=2.14,DCON=32.,TFR=107.,        ,RFIX=1.44,
 NCF=17,CFR=.00025,.0005,.00075,.001,.00125,.0015,.00175,.00225,.003,
 .004,.00525,.007,.0095,.0125,.01575,.020,.025,
 AMC=4.8,3.2,1.6,1.28,.96,.64,.48,.4,.33,.26,.096,.096,.09,.08,.048,.043,.032,
 NST=4*1,UNIT=1.,TF=.6,BF=.6,NSERMO=1,
 CMOT=1.1,
 MST=4*1,POWIN=60.,
 NGUID=1,XGD=40000.,YGD=    0.,VCR=10.,ACR=40.,CONS=4*100.,VGD=87.5,
 U=87.5,
 NP=6,VMAX=.5,VEL=0.,.2,.4,.6,.8,1.,TORQ=-.15,.1,.3,.7,.8,.95,
 RVRA=3600.         ,REMAX=3600.,AST=22.,VOLT=36.,
 LP=2,CUC=1.,22.,RVRB=40.,2.,ROP=4.,
 $END
```

TIME= 1.6999996+01 SECONDS 190 TIME INTERVALS 1SUB INTERVALS

BODY TRANSLATIONAL AND ANGULAR VELOCITIES
8.5946586+01 9.3577059-01 1.9998332+00 2.8074458-03 -4.8483716-03 3.3970750-03

RATIO SIDEWISE VEL TO FORE-AFT VEL 1.0887816-02

INSTATANEOUS HEADING ANGLE DEGREES 2.8768679-01

X,Y,Z TRAJECTORY
1.6665507+03 5.0139189+00 3.4891095+00

DIRECTION COSINE MATRIX

9.9998497-01	-5.0855452-03	2.7689764-03
5.0210985-03	9.9972983-01	2.2737985-02
-2.8836252-03	-2.2723627-02	9.9973758-01

TOTAL ACCELERATIONS BODY AXIS
-4.8189088+00 -3.2233700+01 6.5234255+00 -1.6129106+00 1.6149669-01 1.4891963-01

WHEEL INDICATOR L2, 0 0 0 0

TOTAL ENERGY 1.3111769+04 POWER MOTORS 8.9450550+01 8.9810699+01 9.0890504+01 9.0890523+01

CUMULATIVE DISTANCE TRAVERSED 1.6610952+03

POLAR VECTOR 1.6605583+03 POLAR ANGLE DEGREES 1.7300014-01

WHEEL-GROUND FORCES

I-BODY	J-BODY	K-BODY
-4.5207378+00	-4.7715661+01	7.2495667+01
-7.3843309-01	-1.2664314+00	3.5726553+01
-5.1570310+00	-2.6265571+01	5.5186469+01
-6.0250893+00	-3.3426743+01	6.1332642+01

STEERING ANGLES DEGREES 0.0000000 0.0000000 0.0000000 0.0000000
ACCELERATIONS EXTRA POINTS-BODY AXES

-4.8169088+00	-3.2233700+01	6.5234255+00
-6.9762435+00	-3.3320813+01	-1.5684852+01
-2.6595742+00	-3.3320813+01	3.1089555+01

NON LINEAR INERTIA FORCES BODY AXES 4.156255-02 -9.426063-01 -1.379655+00 3.897612-03 2.139595-02 2.731561-02

GRAVITY FORCES 5.863321-01 4.620472+00 -2.032782+02 0.000000 0.000000 0.000000

WHEEL FORCES -1.644129+01 -1.106744+02 2.247413+02 -2.182982+03 5.426460+02 5.356192+02

BODY HUB COORDINATES UNDER DEFORMATION

	X WHEEL	Y WHEEL	Z WHEEL	ANGLE
1	4.6700457+01	3.5846359+01	-1.6616076+01	1.6621270+02
2	4.6703367+01	-3.5799498+01	-2.0109251+01	1.6332129+02
3	-4.3302371+01	-3.5907354+01	-1.8027974+01	1.7850738+02
4	-4.3303260+01	3.5917724+01	-1.8050362+01	1.7819168+02

C3

BELLCOMM, INC.

Subject: The Dynamics and Performance of a Four-Wheel Vehicle Case 320

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